Study Guide for Benchmark #2
Mt 210 Calculus I
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Window-of-Opportunity: (last week of the semester)

Benchmark testing is the department's way of assuring that students have achieved minimum levels of computational skill. While partial credit is given on the other tests in this course, on the benchmark tests you are expected to demonstrate that you can do basic computations carefully and accurately. Benchmark #2 will cover basic algebraic and derivative computations that we have been using throughout this semester. This benchmark test will have ten problems: one to set up and simplify a difference quotient, and nine on calculating derivatives. *You may <u>not</u> use a calculator or computer while you are working on the benchmark.*

This benchmark will be given during the last 30 minutes of class on DATE (the day we return from Thanksgiving break). To pass the benchmark, you must get 9 or 10 of the problems completely correct; there are no partial credits on the benchmark. If you pass the benchmark on your first attempt, I will record your grade as 100%.

If you do not pass on your first attempt, you may meet with me to go over your mistakes and demonstrate that you have done some additional study. You may retake the benchmark up through DATE_2. The first retake must be done by DATE_1, and the second by DATE_2; you may not retake the benchmark twice in the same day. *Note: I will be giving papers back in class on DATE_1, so if you have not retaken the benchmark before that class period, you may retake it only once.*

If you pass on a retake, your grade will be recorded as the average of your scores on each attempt. If you do not pass the benchmark in three attempts or by DATE_2, your score will be recorded as the <u>lower</u> of 40% or your average on three attempts. In calculating your average in this case, if you have not attempted to retake the Benchmark, your scores for the non-attempt will be considered as 0. So the idea is to do as well as you can on your first attempt, and to make sure that you do pass by DATE_2.

Your grade on this benchmark will contribute 5% to your overall grade for this course. In fact, passing this Benchmark on your first attempt (and earning that 100%) will bring up your current grade in this course. Failing to pass this Benchmark will bring your course grade down by at least half a letter grade.

Setting up a Difference Quotient

You should be able to set up a difference quotient, and simplify it to the point where there is no longer an "h" in the denominator. This is the first step in proving the various short-cut rules for computing derivatives.

- You should be able to set up and simplify a difference quotient for a given function.
- In particular, you should be able to set up and simplify a difference quotient for the following types of functions, where *a* and *b* are constants:
  - $f(x) = ax + b$
  - $f(x) = ax^2 + b$
  - $f(x) = ax^3 + b$
  - $f(x) = ax + b$
  - $f(x) = ax^2 + bx$
  - $f(x) = a$
  - $f(x) = \dfrac{a}{x}$
  - $f(x) = \dfrac{a}{x^2}$
  - $f(x) = a\sqrt{x}$
  - $f(x) = \sqrt{ax}$
  - $f(x) = \sqrt{x + b}$

Calculating derivatives (using the short cuts)

The rules for calculating derivatives are developed throughout Chapters 3, 4, and 5. The formulas are listed in the Summaries of these chapters. In fact, the Chapter 5 Summary gives a cumulative listing of all of the formulas that you need to know for this Benchmark.

You should be able to calculate the derivative of the following kinds of functions:

- Constant function: The graph of a constant function is a horizontal line, so the derivative of a constant function is 0.
- Linear function: The derivative is the slope of the linear function.
- Power function ($f(x) = ax^p$; notice that the variable x is in the base):
  If $f(x) = a\,x^p$, then $f'(x) = ap\,x^{(p-1)}$, where a and p are any numbers.

- exponential function (both e^x and a^x; note that the variable x is in the exponent):
  If $f(x) = e^x$, then $f'(x) = ln(e)\,e^x = e^x$. If $f(x) = e^{g(x)}$, then $f'(x) = ln(e)\,e^{g(x)}\,g'(x)$.
  If $f(x) = a^x$, then $f'(x) = ln(a)\,a^x$. If $f(x) = a^{g(x)}$, then $f'(x) = ln(a)\,a^{g(x)}\,g'(x)$.
  For example, if $f(x) = 5^{3x}$, then $f'(x) = ln(5)\,5^{3x}\,3 = 3\,ln(5)\,5^{3x}$.

- Polynomial function, or the sum or difference of simpler functions: Use the sum (or difference) rules, and take the derivative term-by-term.
- Each of the six trigonometric functions:
  - There are six formulas that you must know. If you recognize the pattern, you just need to memorize three formulas.

- You should be able to calculate the derivative of a simple combination of functions using one of the following strategies:
  - Product rule:

$$\frac{d}{dt}[f(t)\cdot g(t)] = f(t)\cdot g'(t) + g(t)\cdot f'(t)$$

  - Chain rule:

$$\frac{d}{dt}[f(g(t))] = f'(g(t))\cdot g'(t)$$

  - Quotient rule (which is really just a variation on the product rule):

$$\frac{d}{dt}\frac{f(t)}{g(t)} = \frac{g(t)\cdot f'(t) - f(t)\cdot g'(t)}{[g(t)]^2}$$

Answers to Sample Questions for Benchmark #2

The following sample questions will give you an idea of the difficulty level of the questions on this benchmark.

- *There will be one problem asking you to set up and simplify a difference quotient:*

1. Set up a difference quotient for the function $f(x) = 3x^2$, then simplify it to the point where you no longer have Δx (or *h)* in the denominator.
2. Set up and simplify a difference quotient for the function $f(x) = \dfrac{4}{x^2}$.
3. Set up and simplify a difference quotient for the function $f(x) = \sqrt{4x+5}$.

- *There will be nine problems asking you to calculate a derivative:*

4. Find $y'(x)$ if $y(x) = 5x^2 + \pi x + 3.$ Evaluate this derivative at *x* = 2.
5. Find the derivative of $x^2 \cos(x).$
6. Calculate the derivative of $\dfrac{3x^2 + 2x}{\cot(4x)}.$
7. Find $y'(x)$ if $y(x) = 5x^2 + \tan(x) + 3.$
8. Calculate the derivative of $x^2 + 2x.$ Where is this derivative equal to 0?
9. Find the derivative of $x^5 - 5^x.$
10. Find the derivative of $\sec(3x^2).$
11. Calculate the derivative of $e^{3x^2}\sin(5x).$
12. Find the derivative: $3x^2 + \dfrac{4}{x} + 5^{2x} + 2\pi.$
13. Find the derivative of $\dfrac{3x^2 + 2x}{\sin(4x)}.$
14. Find the derivative of $(3x^2 + 2x)\cdot\sin(4x).$

Answers to Sample Questions for Benchmark #2

(Note: I've checked these answers carefully. Let me know if you think I have made an error.)

- *There will be one problem asking you to set up and simplify a difference quotient. It is important to be able to set this up and do the algebraic simplifications correctly.*

1. Set up a difference quotient for the function $f(x) = 3x^2,$ then simplify it to the point where you no longer have Δx (or *h)* in the denominator.

$$\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 3x^2}{h} = \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h} =$$

$$= \frac{h(6x+3h)}{h} = 6x + 3h$$

2. (***Check answer carefully!) Set up and simplify a difference quotient for the function $f(x) = \dfrac{4}{x^2}.$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{4}{(x+h)^2} - \frac{4}{x^2}}{h} = \frac{4x^2 - 4(x+h)^2}{(x+h)^2\, x^2}\,\frac{1}{h}$$

$$= \frac{4x^2 - (4x^2 + 8xh + h^{2)}}{(x+h)^2\, x^2\, h} = \frac{-(8x+h)h}{(x+h)^2\, x^2\, h} = \frac{-(8x+h)}{(x+h)^2\, x^2}$$

3. Set up and simplify a difference quotient for the function $f(x) = \sqrt{4x+5}.$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{4(x+h)+5}-\sqrt{4x+5}}{h}$$
$$= \frac{\sqrt{4(x+h)+5}-\sqrt{4x+5}}{h}\,\frac{\sqrt{4(x+h)+5}+\sqrt{4x+5}}{\sqrt{4(x+h)+5}+\sqrt{4x+5}}$$
$$= \frac{[4(x+h)+5]-(4x+5)}{h\sqrt{4(x+h)+5}+\sqrt{4x+5}} = \frac{4h}{h\sqrt{4(x+h)+5}+\sqrt{4x+5}}$$
$$= \frac{4}{\sqrt{4(x+h)+5}+\sqrt{4x+5}}$$

- *There will be nine problems asking you to calculate a derivative:*

4. Find $y'(x)$ if $y(x) = 5x^2 + \pi x + 3$. Evaluate this derivative at $x = 2$.

$$y'(x) = 10x + \pi + 0$$
$$y'(2) = 10 \cdot 2 + \pi = 20 + \pi$$

5. Find the derivative of $x^2 \cos(x)$.

Use the product rule:

$$x^2 \cdot (-\sin(x)) + 2x \cdot \cos(x) = -x^2 \sin(x) + 2x\cos(x)$$

6. Calculate the derivative of $\dfrac{3x^2+2x}{\cot(4x)}$.

You can do this using either the quotient rule or the product rule. Here it is using the quotient rule:

$$\frac{\cot(4x)\cdot(6x+2) - (3x^2+2x)\cdot(-\csc^2(4x) \cdot 4)}{[\cot(4x)]^2}$$
$$= \frac{\cot(4x)\cdot(6x+2) + 4(3x^2+2x)\cdot\csc^2(4x)}{[\cot(4x)]^2}$$

7. Find $y'(x)$ if $y(x) = 5x^2 + \tan(x) + 3$.

$$y'(x) = 10x + \sec^2(x) + 0$$

8. Calculate the derivative of $x^2 + 2x$. Where is this derivative equal to 0?

$\frac{d}{dx}(x^2+2x) = 2x+2$. This derivative equals 0, when $x = -1$.

9. Find the derivative of $x^5 - 5^x$.

Note that this asks you to use both the power rule and the exponential rule:

$$(x^5 - 5^x)' = 5x^4 - \ln(5)\cdot 5^x$$

10. Find the derivative of $\sec(3x^2)$. Don't forget to apply the chain rule.

$$\sec(3x^2)\tan(3x^2)\cdot 6x = 6x\sec(3x^2)\tan(3x^2)$$

11. Calculate the derivative of $e^{3x^2}\sin(5x)$.

Use the product rule with the chain rule.

$$e^{3x^2}[\cos(5x)\cdot 5] + \sin(5x)\left[e^{3x^2}\cdot 6x\right] = 5e^{3x^2}\cos(5x) + 6x\,e^{3x^2}\sin(5x)$$

12. Find the derivative: $3x^2+\frac{4}{x}+5^{2x}+2\pi$.

$$6x-\frac{4}{x^2}+\ln(5)\cdot\ 5^{2x}\ \cdot 2+0=\ 6x-\frac{4}{x^2}+2\ \ln(5)\ \ 5^{2x}$$

13. (***Sign error!) Find the derivative of $\frac{3x^2+2x}{\sin(4x)}$.

Using the quotient rule: $\frac{\sin(4x)\ \cdot(6x+2)-\ (3x^2+\ 2x)\cdot(-\cos(4x)\cdot 4)}{(\sin(4x))^2}=\frac{\sin(4x)\ \cdot(6x+2)+\ (3x^2+\ 2x)\cdot(4\cos(4x))}{(\sin(4x))^2}$

14. Find the derivative of $(3x^2+2x)\cdot\sin(4x)$.

Using the product rule: $\boldsymbol{(3x^2+2x)\cdot 4\cos(4x)+(6x+2)\cdot\sin(4x)}$